The 2p→1s Pionic Transition†

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ABSTRACT

In an attempt to get better agreement between the theoretical predictions and the observed energies and widths of 2p→1s pion-atomic transitions, we have numerically integrated the Klein-Gordon equation assuming the optical model for the strong interaction potential. We have characterized the interaction by six s-wave and four p-wave parameters. Using the available data from B^{10} to Mg^{24}, we have performed a least squares fit of the s-wave parameters. We find agreement between the observed shifts and our predictions and the real parts of the s-wave parameters agree well with those predicted by the Ericsons. However, our predicted widths vary as much as 50% from the observed widths and find very poor agreement between the imaginary s-wave parameters and the Ericsons' predictions.

I. INTRODUCTION

Pionic 2p→1s transitions in comparatively high Z nuclei such as Na have made it clear that present techniques in predicting the strong interaction widths are not adequate. Originally the standard procedure was to use first order perturbation theory[1] on some assumed nuclear potential. Unfortunately such a method predicts[2] a strong interaction width for the 1s level in Na^{23} which is off by a factor of five. Indeed, whereas Brueckner[3] predicted that the widths should vary roughly as Z^4, there is strong evidence[4,5,6,7] that the widths level off in the region of flourine.

The usefulness of studying the bound pion nuclear system lies in the fact that the pion interacts strongly with both neutrons and protons. If the pion-nuclear interactions can be understood, one has a valuable tool in probing the matter distribution inside a nucleus, not just the charge distribution as in the case of muons. In addition, since the absorption of a pion occurs mainly on two closely correlated nucleons[8], in principle, pionic atoms provide a means of studying such short range correlations.

Two years ago Seki and Cromer[9] and Fulcher, Eisenberg and Le Tourneux[10] contested the use of first order perturbation theory. They showed that one cannot assume that the strong interaction leaves the pion wave function unchanged inside the nucleus. In fact Fulcher et al. proceeded to show that in the 1s level in Na^{23} one might expect a four-fold reduction of the pion overlap with the nucleus.

In addition Trueman[11] pointed out that the interference between the real and imaginary parts of the nuclear potential might appreciably alter the theoretical predictions. Depending upon the relative size of the real and imaginary parts of the potential, the widths might be strongly influenced by the real part of the potential and, to a much lesser degree, the shifts might be affected by the imaginary parts.

Prompted by these theoretical considerations and by the recent experimental activity in pionic x rays, we have modified a computer code to solve the Klein-Gordon equation assuming an optical model for the nucleus. We have numerically integrated the wave equation assuming a functional form for the strong interaction potential discussed by the Ericsons[12]. Their model is based on the premise that one can predict the pion-nuclear interaction from a knowledge of pion-nucleon behavior. It relates the pion-nuclear potential to appropriate pion-nucleon scattering amplitudes and pion absorption rates.

As a result - if the model is correct in concept - one can predict pionic atom binding energies in terms of other experimental quantities, which are reasonably well-known. The most optimistic approach to the problem would be to claim that pionic atom energy levels are completely determined by a sufficiently good knowledge and understanding of pion nucleon scattering processes. On the other hand a more conservative approach would be to admit that the model predicts a functional form for the interaction, which can then be parameterized in terms of a few unknown constants. Our approach was the latter. We have assumed that the Ericson model for the nuclear potential can be expressed in terms of ten constants - six for the s-wave and four for the p-wave scattering amplitudes. We have performed a least squares fit of the six s-wave constants to the experimental $2p \to 1s$ transition energies and widths assuming "reasonable" values for the four p-wave constants, which for the most part have little effect on the 1s strong interaction shift and widths.

In the following section we summarize the relevant theory and define the nuclear scattering parameters. In Section III we summarize the experimental situation. We discuss the sensitivity of our results to the assumed charge distribution in Section IV. Then we discuss the predicted widths and shifts. Finally in Section VI we examine the best fit s-wave nuclear parameters obtained from the pionic x-ray data and compare these to those predicted by the Ericsons.

II. THEORY

For non-relativistic pionic atoms one can use the Schrodinger equation

$$\left(\frac{p^2}{2m} + V\right) \psi = \varepsilon\psi, \tag{1}$$

where V is the sum of the electromagnetic potential V_c and the nuclear potential V_n, ε is the binding energy of the pion, and m and p are the pion mass and momentum respectively. However, anticipating the relativistic corrections yielded by the Klein-Gordon equation relative to the Schrodinger equation, one finds in Na^{23} the relativistic shifts are nearly 3 keV, much larger than the experimental error. Clearly a relativistic generalization of the above is necessary.

One procedure as discussed in Goldberger and Watson[13] is to replace the non-relativistic kinetic energy operator by its relativistic generalization $(\sqrt{p^2 + m^2} - m)$. One can then obtain a relativistic wave equation involving a commutator

$$(p^2 + m^2)\ \psi + [(p^2 + m^2)^{1/2},V]\ \psi = (E - V)^2\ \psi, \tag{2}$$

where E is the total pion energy including the its rest mass. With the exception of the commutator, this equation is identical to the Klein-Gordon equation, assuming, of course, that some suitable relativistic potential can be found. To first order in p^2, the expectation value of the commutator vanishes for a real potential. We chose to treat the Coulomb part of the problem relativistically, and to first order the commutator will not contribute to this part of the interaction. Since we intended to use an optical potential for the strong interaction and this potential is non-relativistic, we kept only those terms in the wave equation which are linear in V_n and dropped the commutator term because it is a relativistic correction. When we took into consideration the finite mass of the nucleus, our wave equation became:

$$\left(1 + \frac{m}{AM}\right) \nabla^2\psi = \left(m^2 - (E - V_c)^2 + 2\, m\, V_n\right)\psi \tag{3}$$

where M is the nucleon mass and A is the atomic mass number. Letting $E = m + \varepsilon$, we find

$$\frac{\nabla^2}{2\mu}\, \psi + \left[\varepsilon - V + \frac{(\varepsilon - V_c)^2}{2m}\right] \psi = 0 \tag{4}$$

where μ is the reduced mass of the pion-nuclear system. In Eq. (4) we can clearly see the relativistic correction $\frac{(\varepsilon - V_c)^2}{2m}$ due to the electromagnetic interaction.

According to the Ericsons one can describe the pion nuclear interaction using the following non-relativistic potential V_n

$$V_n = V_L - \frac{1}{2m} \nabla\alpha\nabla \tag{5}$$

where the local potential V_L, which corresponds to s-wave pion nucleon interactions, can be written

$$V_L = \frac{-m_1(r) - m_2(r)}{2\mu} \tag{6}$$

and the non-local potential, which corresponds to p-wave pion nucleon interaction, is given in terms of α which in turn can be written

$$2\mu\alpha = -L(r)\ (n_1(r) + n_2(r)). \tag{7}$$

Here $m_1(r)$ and $n_1(r)$ correspond to single nucleon processes; $m_2(r)$ and $n_2(r)$ are two-nucleon contributions. The function $L(r)$ takes into account short-range correlations between scatterers and is analogous to the Lorentz-Lorenz effect which arises from the scattering of light in dense classical media[12]. A potential of this form is often referred to as a Kisslinger[14] potential. These functions are given by

$$m_1(r) = \frac{4\pi}{m} (1 + m/M)(b_0 + b_1(N - Z)/A)\rho(r) + \Delta m_{1F}(r) + \Delta m_C(r), \qquad (8)$$

$$m_2(r) = \frac{4\pi}{m^4} \left(1 + \frac{m}{2M} \right)\left(B_0 + \frac{B_1}{A-1}\right)\rho^2(r) + \Delta m_{2F}(r), \qquad (9)$$

$$L(r) = \left(1 + \frac{(n_1(r) + n_2(r))}{3}\right)^{-1}, \qquad (10)$$

$$n_1(r) = \frac{4\pi}{m^3} (1 + m/M)(c_0 + c_1(N - Z)/A)\rho(r), \qquad (11)$$

$$n_2(r) = \frac{4\pi}{m^6} \left(1 + \frac{m}{2M}\right) C_0\, \rho^2 (r), \qquad (12)$$

where $\rho(r)$ is the nucleon density distribution normalized to A (we are assuming here that the proton and neutron distributions are identical).

The finite correlation length correction $\Delta m_C(r)$ is given by

$$\Delta m_C(r) = \frac{-9\pi^2}{m^2}(1 + m/M)^2\rho^2(r)(b_0{}^2 + b_1{}^2)\, p_F{}^{-2}, \qquad (13)$$

where p_F is the Fermi momentum of a nucleon in the nucleus and is of the order of 250 MeV/c.

The corrections Δm_{1F} and Δm_{2F} due to the Fermi motion of the nucleons inside the nucleus are given by

$$\Delta m_{1F} = \left(\frac{m}{M}\right)^2 n_1(r)\, L(r) < p_N^2 > \qquad (14)$$

$$\Delta m_{2F} = \left(\frac{m}{2M}\right)^2 n_2(r)\, L(r) < p_{2N}^2 > \qquad (15)$$

where $<p_N^2>$ is the mean square momentum of a single nucleon and is of the order of three-fifths $p^2{}_F$; $<p_{2N}^2>$ is the mean square momentum of a pair of nucleons and is of the order of twice $<p_N^2>$.

The single nucleon parameters b_0, b_1, c_0 and c_1 are real; whereas, the two nucleon parameters B_0, B_1 and C_0 are complex.

In solving Eq. 4 numerically we found it useful to rewrite the equation in the following form:

$$\nabla \cdot \frac{(1+\alpha)}{2\mu} \nabla\psi + [(\varepsilon - V_c)(1 + \frac{\varepsilon - V_c)}{2m} - V_L]\ \psi = 0.$$

The code used to solve the Klein-Gordon equation with complex potential was based on methods used in a program by McKee[15], which solves the Dirac equation for muonic atoms. The Klein-Gordon equation is solved exactly using a Runge-Kutte method for developing the wave functions. A general discussion of the principles of such techniques is given by Blatt[15]. We perform a perturbation calculation to evaluate the vacuum polarization and Lamb shift corrections.

To check the validity of the integration procedure we calculated the binding energy of a 1s pion in Na^{23} for a point Coulomb potential and compared this with the Klein-Gordon point value. The agreement was within 0.05 keV, which is consistent with our convergence criterion.

In this work, we have assumed that the c's - the non-local parameters- are fairly well-known and used essentially the Ericsons' estimates. We then used all available 1s shift and width data from B^{10} to Mg to determine the six parameters b_0, b_1, ReB_0, ImB_0, ReB_1 and ImB_1. The values so obtained can then be compared to the values predicted by the Ericsons. Experimental data is available for He^4, Li^6, Li^7 and Be^9 but we hesitate to apply the optical model to systems with so few nucleons where surface effects and A^{-1} terms may become important. We have, however, made predictions for these elements although they have not been used in obtaining best values for the s-wave parameters. We have also determined the s-wave parameters assuming that the corrections $\Delta m_{1F}(r)$, $\Delta m_{2F}(r)$ and Δm_C are all zero. This was done to avoid compounding the inherent uncertainties of these corrections with the uncertainties in our determination of the s-wave parameters.

III. EXPERIMENTAL DATA

The transition energies and widths for the 2p→1s transitions have been measured by many groups. The first measurements using NaI scintillation counters and proportional counters have been summarized by West[17]. These measurements disagree with each other. More recently solid-state-detectors have been used and these data are in good agreement with the later measurements of West except for the width of the Be^9 line which was not analyzed properly[4].

In our analysis, we have taken only the solid-state-detector measurements because of their higher resolution in this energy range. The higher resolution not only allows more accurate measurement but it also helps to separate out the the muonic and nuclear gamma transitions in the spectra which are close in energy to the pionic lines and which complicate the analysis of these lines.

Table I summarizes the present data on pionic widths and shifts for the 2p→1s transitions as measured by solid-state-detectors. An average value for each isotope has been derived by weighting each measurement with the inverse square of its quoted error. The error on the average value is the reciprocal of the summed inverse squares of the error for each measurement. The saturation effect, mentioned in the introduction, is clearly seen by inspecting the averaged widths as a function of Z.

IV. CHARGE PARAMETERS

In general for the nuclei examined ($Z \leq 12$) it is sufficient - to a good approximation - to describe the charge density in terms of a single length parameter. This is indeed fortunate, since the available charge distribution information generally yields very accurately only a single length parameter, which can be related to the rms radius. The rms radii have been taken from electron scattering and muonic x-ray data[18,19]. Whenever data were available from both sources, the results of the more accurate determination was used. A summary of the relevant charge parameters is found in Table II.

In general we found that the shifts and widths were insensitive to the detailed charge distribution as long as the mean square radius was held constant. For the lower Z nuclei both shell model distributions and Fermi distributions (labeled G and F respectively in Table II) were considered. For nuclei with $Z \leq 8$ the shell model distribution is a Gaussian with a central depression and is given by

$$\rho = \rho_0 \left(1 + \frac{wr^2}{c^2}\right) \exp\left(\frac{-r^2}{c^2}\right) \tag{17}$$

where c is the characteristic length parameter; w is the central depression parameter given by

$$w = (Z - 2)/c;$$

and the root mean square radius r_{rms} is given by

$$r^2_{rms} = c^2\,(6 + 15w)/(4 + 6w);$$

ρ_0 is a normalization constant.

The Fermi distribution can be written in the following convenient form:

$$\rho = \rho_0 \left(1 + \exp\left(n\left(\frac{r}{c} - 1\right)\right)\right)^{-1} \tag{18}$$

where the dimensionless parameter n is related to the half-density radius c and the skin thickness (90% - 10%) t through the expression

$$n = c/(0.228t).$$

Comparison of the energies and widths for $Z \leq 8$ nuclei for shell model and Fermi distributions with the same r_{rms} were made. The sensitivity to the change of distribution is negligible. For example for O^{16} we assumed a Gaussian distribution with $c/A^{1/3} = 0.70$ F and w = 2 giving the same value of $R_{eq}/A^{1/3}$ shown in Table II for a Fermi distribution and found that the transition energy and the width differed by 0.06 keV from the results obtained from a Fermi distribution. In our final calculations we assumed a shell model distribution for $Z \leq 4$ and a Fermi distribution for the other cases.

In a two parameter distribution like a Fermi distribution, r_{rms} does not determine the half-density radius and skin thickness uniquely. Therefore we had to check the sensitivity of the binding energies and widths to the particular combination of c and n used. As an example a comparison of the results of two pairs of c and n is given in Table III. Again the results are insensitive to the change in charge distribution. Here we have assumed a change in c of 2.5%. The effect is no more than a 0.23 keV change in the width of the 1s level and an even smaller change in the transition energy. For the most part we have chosen the values of c given by Elton[19], which are generally known to within 2% and determined n from the observed r_{rms}.

It should be noted that muonic x rays and electron scattering yield information concerning the nuclear charge distribution with the finite size of the proton folded into the distribution. To get the density of nucleon centers we subtracted out the proton radius from the mean square nuclear radius, that is

$$< r^2_M > = < r^2_C > - < r^2_p > \tag{19}$$

where $< r^2_M >$ is the mean square radius of the density of nucleon centers;

$< r^2_C >$ is the nuclear mean square charge radius determined by muonic x rays or electron scattering;

$< r^2_p >^{1/2}$ is the root mean square proton charge radius, observed to be 0.776 F[20].

It should be stressed that the predictions were quite sensitive to whether we included this last correction or not as the finite size of the proton constitutes an appreciable portion of the nuclear volume even in Na^{23}. In this element $< r^2_C >^{1/2}$ is 2.945 F. whereas $< r^2_M >^{1/2}$ is 2.840 F. As a result the width would be 1 keV less and the transition energy would be 0.4 keV more if the proton size were not removed.

Throughout our discussion we have assumed that the neutrons are described by the same distribution as the protons. The effect of letting the neutron distribution vary relative to the proton distribution bears future investigation.

V. RESULTS

We shall now discuss our solutions using the nuclear parameters obtained from the least squares fit of the predicted widths and shifts to the data. In the next section we shall examine the values of the nuclear parameters so obtained.

In Fig. 1 we display the relative strengths of the local strong potential in comparison with the electromagnetic potential for a point as well as a finite charge distribution for a pion in Na^{23}. It is interesting to note that at the center of the nucleus the repulsion due to the real part of the local potential is four times stronger than the Coulomb interaction. Of course, the total strong potential is not 26 MeV since the non-local term is not included in Fig. 1. However the net strong potential is repulsive and it is this strong repulsion which reduces the overlap of the pion wave function with the nucleus, illustrated in Fig. 2. We have found that inside the Na^{23} nucleus the probability density of the pion has been reduced to 24% of the point nucleus value in excellent agreement with Fulcher et al[10]. For convenience we have also included in Fig. 2 a plot of the pion probability density for a point Coulomb interaction and a finite Coulomb interaction.

In Table IV we compare our predictions including all corrections listed in the theoretical discussion with the experimentally observed widths and energies of $2p \rightarrow 1s$ pionic transitions. If we exclude the corrections Δm_{1F}, Δm_{2F}, and Δm_C, we obtain different "effective" nuclear parameters but our predicted transition energies and widths are not appreciably changed. Quantitatively there is excellent agreement between prediction and experiment as far as the transition energies are concerned but we have not been able to duplicate the strange behavior of the widths in the region of Na^{23}. The least squares fit program has produced better agreement in Na^{23} at the expense of O^{16} and O^{18}. In order to get a reasonably small width in Na^{23} we have had to accept fairly large underestimations in oxygen. Even the sign of the isotope shift in oxygen is not correctly predicted suggesting some weakness in the formalism in handling the iso-spin dependence. However, in general, there has been a significant improvement in the predictions through the use of an exact solution of the Klein-Gordon equation compared to those obtained with first order perturbation theory[2].

VI. OPTICAL POTENTIAL PARAMETERS

In our present analysis we concentrate on the parameters that characterize the local part of the pion-nuclear optical potential. This is a reasonable simplification in our case since we are presently concerned with an analysis of the available experimental data on $2p \to 1s$ transitions in nuclei with $Z < 13$. For such nuclei the local interaction dominates.

For the non-local potential we used the theoretical parameters estimated by the Ericsons[12]. However, since these parameters are subject to uncertainties, we have studied the effects of variations of these parameters on our results. The influence of the non-local potential is expected to be most pronounced in the heavier nuclei; therefore, we consider Na^{23}. Varying c_0 within a range of twice its uncertainty produced a change in our calculated transition energy of about 1/2% and a change in our calculated width of about 3%. The influence of c_1 was more than an order of magnitude down from that of c_0. Since the complex parameter C_0 is not known to the same precision as c_0 we allowed this parameter to vary within a range of 50% of the values given by the Ericsons. Typical changes in the calculated energy were again about 1/2% whereas changes in the calculated widths were about 15%. The non-local parameters were also varied in the case of C^{12}. The effect on the transition energy was negligible. In the case of the width we found variations up to about 5%. Thus the effects due to the non-local potential, which certainly are not insignificant, are small enough that they will not seriously affect our present results.

With the non-local parameters given in Table V we have performed a least squares analysis based on the available experimental data in order to determine a set of best fit parameters that characterize the local potential. In Table V we give those s-wave parameters obtained by including all the corrections mentioned in Section II. We have also calculated effective s-wave parameters, which would be obtained if the corrections in Eqs. 9, 10 and 11 were set to zero. In both cases we include the predictions made by the Ericsons. The prediction for B_1 was estimated following arguments given in Ref. 12, Appendix B. The origin of this term is due to spin and isospin dependent parts of the two nucleon optical potential. To estimate its magnitude we averaged over the spin and isospin variables according to methods discussed in Goldberger and Watson[13].

It is seen from Table V that the values of b_0, b_1, ReB_0, and ReB_1 obtained with and without corrections agree reasonably well with those predicted by the Ericsons. Furthermore, the calculated transition energies are in agreement with those observed experimentally. This supports the point of view of Ref. 12 that the pion-nuclear interaction can be understood in terms of the basic pion-nucleon interactions, at least in the case of an elastic process (energy shift).

The calculated widths are still in fairly poor agreement with those measured experimentally. This lack of agreement is reflected in the determination of the imaginary parts of our best fit optical potential parameters. These parameters disagree strongly with the corresponding theoretical predictions. It should be stressed, however, that if one were to leave out F^{19} and Na^{23} in the determination of the local parameters there would be much better agreement in the widths of the remaining nuclei. Whether our lack of agreement is a reflection of a weakness in the optical potential formalism or is related to some specific effect in Na^{23} and F^{19} is not clear at this time.

This problem has also been examined by Backenstoss et al.[21] where the optical potential has been characterized by fewer parameters. They reach similar conclusions to ours.

VII. CONCLUSIONS

We have found that most of the discrepancy between earlier predictions of pion-atomic transition energies and widths was due to not considering the distortion of the pionic wave function by the strong interaction and the interference of the real and imaginary parts of the nuclear potential. We have found quite good agreement between predicted and observed transition energies and acceptable agreement between the real part of the nuclear potential inferred from the pionic-atomic data with the predictions made by the Ericsons. However, there is still a residual inconsistency between prediction and experiment with regard to the widths. It is not clear whether this latter disagreement is due to a weakness in the imaginary part of the optical potential formalism or is related to some effect in the higher Z nuclei which has not been taken into consideration.

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TABLE I

SUMMARY OF DATA FOR PIONIC 2p→1s TRANSITIONS

Isotope	2p→1s Transition Energies				Widths			
	Berkeley[a]	CERN[b]	W&M[c]	Average	Berkeley[a]	CERN[b]	W&M[c]	Average
He^4	---	---	10.69±0.06	10.69 ±0.06	---	---	0.000±0.086	0.000±0.086
Li^6	23.9±0.2	---	24.18±0.06	24.157±0.057	0.39±0.36	---	0.15 ±0.05	0.155±0.050
Li^7	23.8±0.2	---	24.06±0.06	24.038±0.057	0.57±0.30	---	0.19 ±0.05	0.200±0.049
Be^9	42.1±0.2	42.38±0.20	42.32±0.05	42.311±0.047	0.85±0.28	1.07±0.30	0.58 ±0.05	0.601±0.049
B^{10}	64.9±0.2	65.94±0.18	65.79±0.11	65.663±0.085	1.4 ±0.5	1.27±0.25	1.68 ±0.12	1.594±0.106
B^{11}	64.5±0.2	64.98±0.18	65.00±0.11	64.905±0.085	2.3 ±0.5	1.87±0.25	1.72 ±0.15	1.793±0.125
C^{12}	93.3±0.5	92.94±0.15	93.19±0.12	93.099±0.092	2.6 ±0.5	2.96±0.25	3.25 ±0.15	3.138±0.125
N^{14}	123.9±0.5	124.74±0.15	---	124.671±0.144	4.1 ±0.4	4.48±0.30	---	4.343±0.240
O^{16}	160.6±0.7	159.95±0.25	---	160.022±0.235	9.0 ±2.0	7.56±0.50	---	7.645±0.485
O^{18}	---	155.01±0.25	---	155.01 ±0.25	---	8.67±0.70	---	8.67 ±0.70
F^{19}	196.5±0.5	195.9 ±0.5	---	196.20 ±0.35	4.6 ±2.0	9.4 ±1.5	---	7.67 ±1.20
Na^{23}	277.2±1.0	276.2 ±1.0	277.7 ±0.5	277.37 ±0.41	4.6 ±3.0	10.3 ±4.0	6.2 ±1.2	6.29 ±1.07
Mg	330.3±1.0	---	---	330.3 ±1.0	---	---	---	---

a. Ref. 4 and 5

b. Ref. 6

c. Ref. 2 and 7

TABLE II

Charge Parameters

Z	A	$\frac{c}{A^{1/3}}$ (F)	n	w	$\frac{R_{eq}}{A^{1/3}}$ (F)	Distribution
2	4	0.825	-	0.0	1.304[a]	G
3	6	1.026	-	0.333	1.794[a]	G
3	7	0.930	-	0.333	1.625[a]	G
4	9	0.856	-	0.667	1.562[a]	G
5	10	0.91	3.79	-	1.467[a]	F
5	11	0.87	3.79	-	1.403[a]	F
6	12	1.00[a]	5.18	-	1.363[b]	F
7	14	1.01	5.8	-	1.311[b]	F
8	16	1.03[a]	5.59	-	1.357[b]	F
8	18	1.034	5.59	-	1.366[b]	F
9	19	1.03	5.40	-	1.368[b]	F
11	23	1.03	5.80	-	1.337[b]	F
12	24	1.01[a]	5.42	-	1.349[b]	F

[a] Ref. 18

[b] Ref. 19

TABLE III

EFFECTS OF CHANGES IN CHARGE DISTRIBUTION IN Na^{23}

$\frac{c}{A^{1/3}}$ (F)	n	$\frac{R_{eq}}{A^{1/3}}$ (F)	Transition Energy (keV)	Width (keV)
1.03	5.8	1.337	278.36	10.70
1.005	5.47	1.336	278.43	10.93

TABLE IV

COMPARISON OF EXPERIMENTAL AND PREDICTED

PIONIC $2p \rightarrow 1s$ TRANSITIONS

Isotope	$2p \rightarrow 1s$ Transition Energies Experiment	Prediction[a]	1s Widths Experiment	Prediction[a]
He^4	10.69 ±0.06	10.63	0.000±0.086	0.23
Li^6	24.157±0.057	24.15	0.155±0.050	0.32
Li^7	24.038±0.057	23.93	0.200±0.049	0.31
Be^9	42.311±0.047	42.14	0.601±0.049	0.70
B^{10}	65.663±0.085	65.73	1.594±0.106	1.79
B^{11}	64.905±0.085	64.90	1.793±0.125	1.73
C^{12}	93.099±0.092	93.15	3.138±0.125	3.08
N^{14}	124.671±0.144	124.53	4.343±0.240	4.75
O^{16}	160.022±0.235	159.87	7.645±0.485	5.88
O^{18}	155.01 ±0.25	155.09	8.67 ±0.70	4.67
F^{19}	196.20 ±0.35	195.01	7.67 ±1.20	6.49
Na^{23}	277.37 ±0.41	278.36	6.29 ±1.07	10.70
Mg^{24}	330.3 ±1.0	329.84	---	13.83

[a]Using best fit nuclear parameters in Table V.

TABLE V

S-WAVE NUCLEAR PARAMETERS

	Best Fit From Pionic X rays	Ericsons[a] Predictions
b_0	-0.023 ± 0.006	-0.012 ± 0.004
b_1	-0.117 ± 0.010	-0.097 ± 0.007
$\mathrm{Re}B_0$	-0.016 ± 0.021	-0.01
$\mathrm{Im}B_0$	0.0005 ± 0.0047	0.012 ± 0.001
$\mathrm{Re}B_1$	-0.090 ± 0.060	-0.10
$\mathrm{Im}B_1$	0.466 ± 0.058	0.099 ± 0.022

EFFECTIVE S-WAVE NUCLEAR PARAMETERS
(Without Corrections)

	From Pionic X rays	Ericsons[a] Predictions
b_0	-0.016 ± 0.006	-0.008 ± 0.004
b_1	-0.111 ± 0.010	0.097 ± 0.007
$\mathrm{Re}B_0$	-0.051 ± 0.021	-0.034 ± 0.004
$\mathrm{Im}B_0$	0.0021 ± 0.0047	0.012 ± 0.001
$\mathrm{Re}B_1$	-0.090 ± 0.060	-0.10
$\mathrm{Im}B_1$	0.466 ± 0.058	0.099 ± 0.022

ASSUMED P-WAVE PARAMETERS

c_0	0.21
c_1	0.18
$\mathrm{Re}C_0$	-0.1
$\mathrm{Im}C_0$	0.1

[a] All the parameters except B_1, which is discussed in Section VI, are taken from Ref. 12.

Figure Captions

Fig. 1. Pionic nuclear potentials in Na^{23}. The real and imaginary parts of the local strong potential are compared to the electromagnetic potentials due to point and finite charge distributions.

Fig. 2. The 1s pion probability density in Na^{23} assuming (a) a point Coulomb charge distribution, (b) a finite Coulomb charge distribution (both with no strong interaction) and (c) a finite Coulomb and strong interaction. The curves are normalized so that the total probability of the pion (the probability density integrated over all space) is the same for the three cases shown.

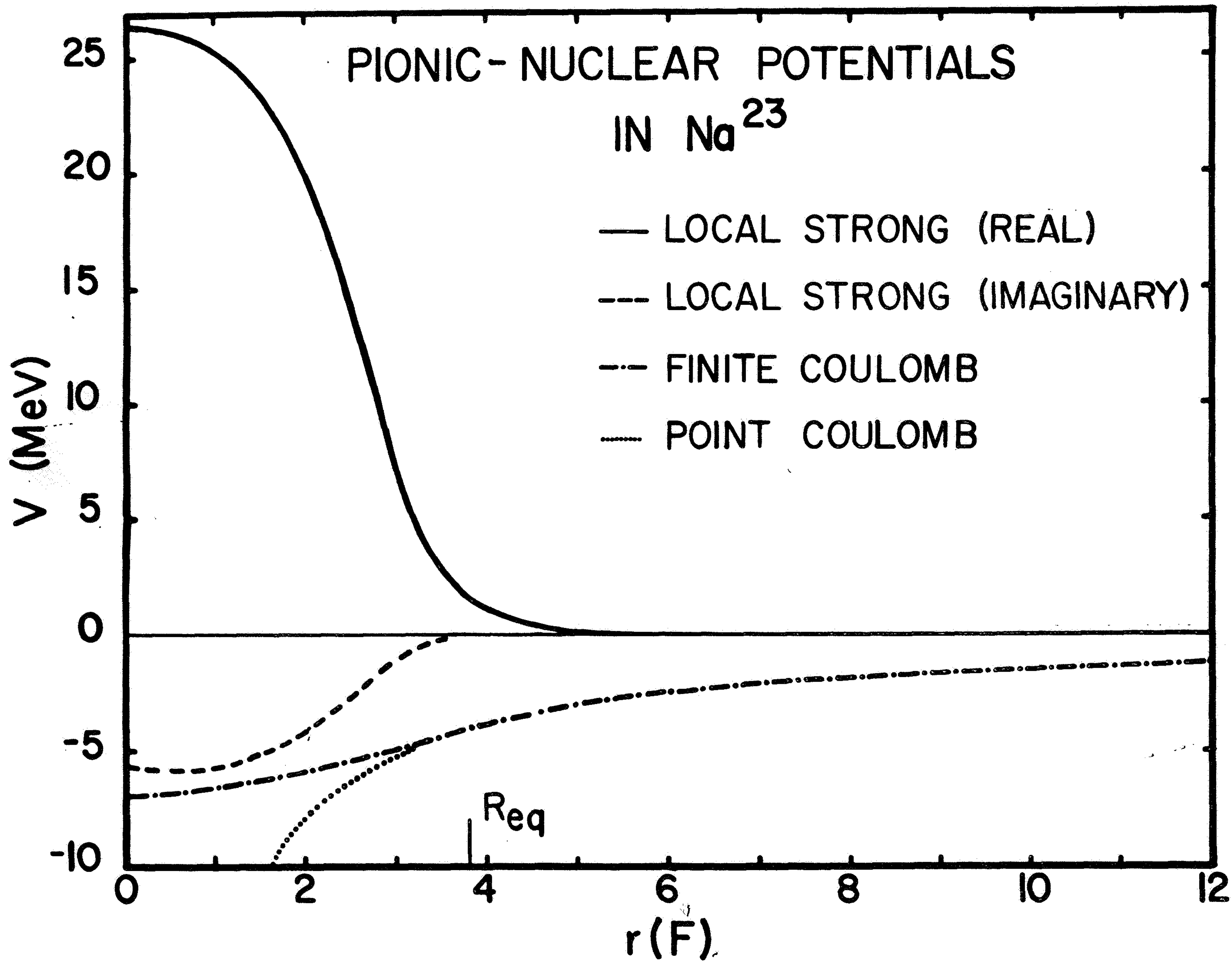
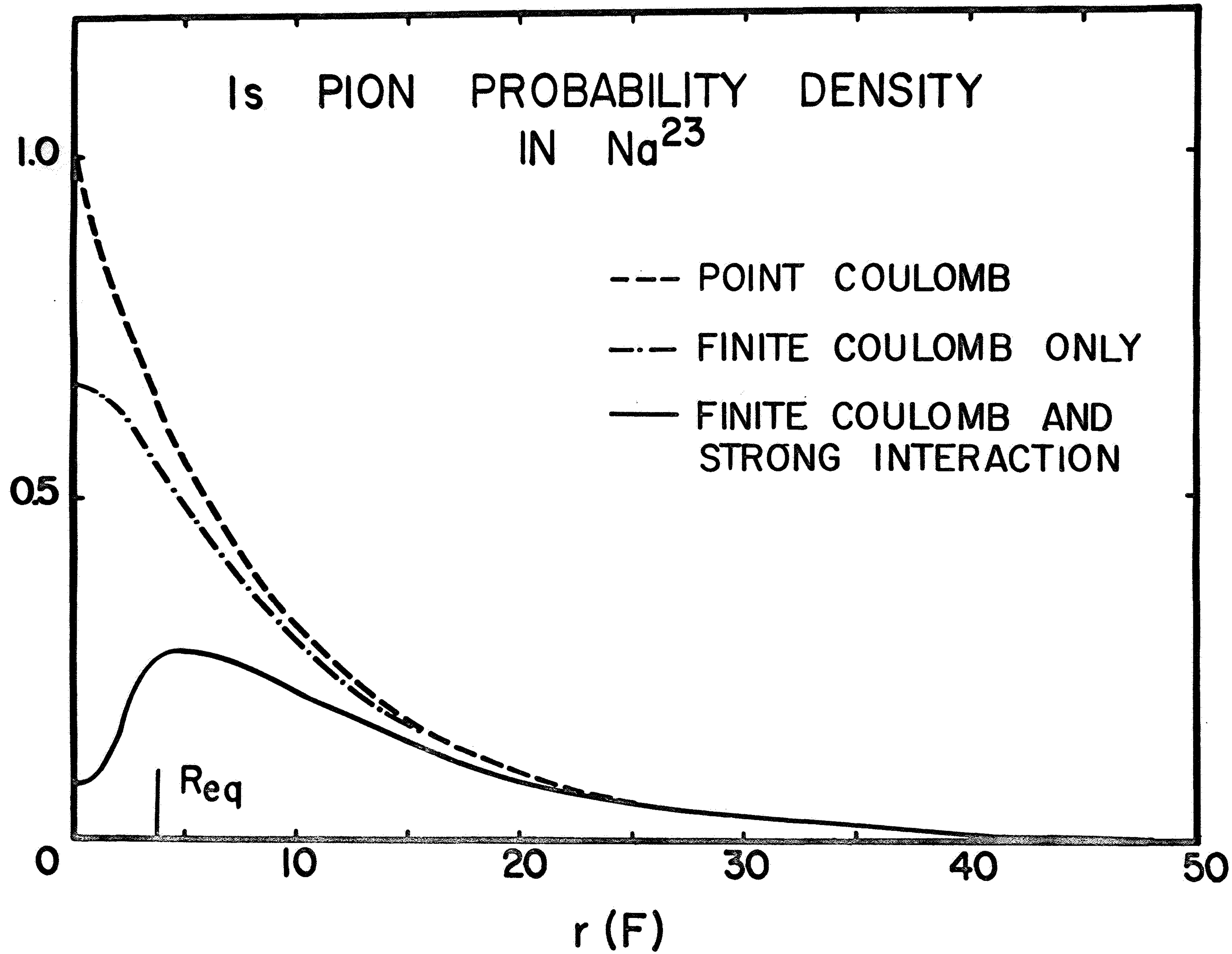

PIONIC-NUCLEAR POTENTIALS
IN Na^{23}
LOCAL STRONG (REAL)
LOCAL STRONG (IMAGINARY)
FINITE COULOMB
POINT COULOMB
V (MeV)
25
20
15
10
5
0
-5
-10
0
2
4
6
8
10
12
r (F)
R_{eq}

Fig. 1



1s PION PROBABILITY DENSITY
IN Na23
POINT COULOMB
FINITE COULOMB ONLY
FINITE COULOMB AND
STRONG INTERACTION
1.0
0.5
0
10
20
30
40
50
Req
r (F)

FIG. 2